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Algebras of Functions on Compact Quantum Groups, Schubert Cells and Quantum Tori

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Abstract. The structures of Poisson Lie groups on a simple compact group are parametrized by pairs (a, u), where $a \in R$, $u \in \Lambda^2 \mathfrak{h}_R$, and $\mathfrak{h}_R$ is a real Cartan subalgebra of complexification of Lie algebra of the group in question. In the present article the description of the symplectic leaves for all pairs (a, u) is given. Also, the corresponding quantized algebras of functions are constructed and their irreducible representations are described. In the course of investigation Schubert cells and quantum tori appear. At the end of the article the quantum analog of the Weyl group is constructed and some of its applications, among them the formula for the universal R-matrix, are given.

Introduction

0.1. Let G be a finite-dimensional simply connected simple complex Lie group. G is called a Poisson Lie group ([3]) if it is a Poisson manifold and the multiplication $\mu: G \times G \to G$ is a morphism of Poisson manifolds. There exist many Poisson Lie group structures on the fixed group G. All of them are listed in [1].

Let $K \subset G$ be a maximal compact subgroup. The results of [1] imply (see [28, 29] and Sect. 1 below) that all Poisson Lie group structures on K are parameterized essentially by pairs (a, u), where a is a real number, $u \in \Lambda^2 \mathfrak{h}_{\mathbb{R}}$ and $\mathfrak{h}_{\mathbb{R}}$ is a real Cartan subalgebra of the Lie algebra $g = \mathrm{Lie}\, G$.

Fix pair (a, u). The corresponding Poisson Lie group is denoted by $K(a, u)$. In Drinfeld's theory [3], the group $K(a, u)$ may be viewed as a classical object that is subject to quantization. To formulate this in the language of algebras of functions, consider the algebra of regular functions $\mathbb{C}[G]$ on a Poisson Lie group G. This is a Poisson Hopf algebra in the sense of [3], i.e. the comultiplication $\delta: \mathbb{C}[G] \to \mathbb{C}[G] \otimes \mathbb{C}[G]$ is a Poisson algebra homomorphism. Consider the algebra of K-finite functions $\mathbb{C}[K]$ consisting of restrictions $f|_K$, where $f \in \mathbb{C}[G]$. The algebra $\mathbb{C}[K]$ has involution (complex conjugation $f \mapsto \bar{f}$), and the comultiplication $\delta: \mathbb{C}[K] \to \mathbb{C}[K] \otimes \mathbb{C}[K]$ is a homomorphism of algebras with involution