Commun. Math. Phys. 138, 487–505 (1991)

Communications in Mathematical Physics

Low Temperature Properties of the Hierarchical Classical Vector Model

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Received April 20, 1990; in revised form December 18, 1990

Abstract. We obtain low temperature properties of the classical vector model in a hierarchical formulation in three or more dimensions. We consider the lattice model in a zero or non-zero magnetic field, where the single site spin variable $\phi \in R^\nu$ has a density proportional to $e^{-\lambda(\phi^2-1)^2}$ for large $\lambda \leqq \infty$. Using renormalization group methods we obtain a convergent expansion for the free energy with zero magnetic field. For non-zero fields a shift formula is used to obtain the effective action generated by the renormalization group transformation (RGT). To obtain the pure state zero field free energy and spontaneous magnetization we take the thermodynamic limit together with the zero field limit at a specified rate. The spontaneous magnetization, m, is calculated, is non-zero and the pure state free energy coincides, as expected, with the zero field free energy. Also the sequence of zero field actions does not have a limit but we show that the sequence of actions generated from the original action shifted by m does; the limiting action corresponds to a non-canonical Gaussian fixed point of the RGT.

I. Introduction and Results

Consider the d-dimensional lattice classical vector model with partition function given by

$$Z = \int e^{\beta[1/2(\phi, \Delta\phi) + (h, \phi_1)]} \pi\delta(|\phi(x)|^2 - 1) d\phi(x), \tag{1.1}$$

where $\phi(x) = (\phi_1(x), \ldots, \phi_\nu(x)) \in R^\nu$ and Δ is the lattice Laplacian. We want to obtain low temperature (large β) properties of a hierarchical formulation of this model. Formal high and low expansions have been obtained for physical quantities such as the free energy, magnetization and correlation functions for this model [1, 2]. Rigorous low temperature results have been obtained in [3, 4] for $d=2$ and in [5] for $d=3$, $\nu=2$. For $d=3$, $\nu=2$ there is spontaneous magnetization (see [6]) and the truncated correlation functions for $h=0$ are expected, according to the Goldstone picture, to exhibit canonical $|x-y|^{-(d-2)}$ falloff perpendicular to