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Inverse Backscattering in Two Dimensions*

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Abstract. This article extends the authors' previous results (Commun. Math. Phys. **124**, 169–215 (1989) to inverse scattering in two space dimensions. The new problem in two dimensions is the behavior of the backscattering amplitude near zero energy. Generically, this has the form

$$a(\xi/|\xi|, -\xi/|\xi|, |\xi|) = 2\pi(2\pi\beta + \ln|\xi|)^{-1} + b(\xi),$$

where $b(0) = 0$ and $b(\xi)$ is Hölder continuous. In order to work in weighted Hölder spaces as before, the constant β and the function $b(\xi)$ must now be interpreted as "coordinates" on the space of backscattering data. In this setting the mapping to backscattering data is again a local diffeomorphism at a dense open set in the real-valued potentials.

I. Introduction

This article extends the results of [ER] to problems in two space dimensions. As in [ER] we define the scattering amplitude associated with the Hamiltonian $-\Delta + q(x), x \in \mathbb{R}^n$, in the following way. Let $h(\xi, \zeta, k), (\xi, \zeta, k) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+$, be the solution to the integral equation

$$h(\xi, \zeta, k) + (2\pi)^{-n} \int_{\mathbb{R}^n} \frac{\hat{q}(\xi - \eta) h(\eta, \zeta, k)}{|\eta|^2 - (k + i0)^2} d\eta = -\hat{q}(\xi - \zeta), \tag{I.1}$$

where $\hat{q}$ denotes the Fourier transform

$$\hat{q}(\xi) = \int_{\mathbb{R}^n} e^{-ix\cdot\xi} q(x) dx.$$

Then the scattering amplitude is the restriction of h to $|\xi| = |\zeta| = k$, and the backscattering amplitude is $h(\xi, -\xi, |\xi|)$, $\xi \in \mathbb{R}^n \backslash 0$. We assume that $\hat{q}$ belongs to one of the weighted Hölder spaces, $H_{\alpha,N}$ used in [ER]. The space $H_{\alpha,N}$ is defined as the closure of $C_0^\infty(\mathbb{R}^n)$ in the norm

$$\|f\|_{\alpha,N} = \|\Lambda^N f\|_\alpha,$$

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