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$(Z_N \times)^{n-1}$ Generalization of the Chiral Potts Model

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Abstract. We show that the R-matrix which intertwines two n-by-N^{n-1} state cyclic L-operators related with a generalization of $U_q(sl(n))$ algebra can be considered as a Boltzmann weight of four-spin box for a lattice model with two-spin interaction just as the R-matrix of the checkerboard chiral Potts model. The rapidity variables lie on the algebraic curve of the genus $g=N^{2(n-1)}((n-1)N-n)+1$ defined by $2n-3$ independent moduli. This curve is a natural generalization of the curve which appeared in the chiral Potts model. Factorization properties of the L-operator and its connection to the SOS models are also discussed.

0. Introduction

As it is observed in [1] the chiral Potts model [2–4] can be considered as a part of some new algebraic structure related to the six vertex R-matrix. In particular, the high genus algebraic relations among the Boltzmann weights of the chiral Potts model arise as a condition of the existence of an intertwining operator for two different representations of some quadratic Hopf algebra [5–7] which generalizes the $U_q(sl(2))$ algebra.

It is natural to make an attempt to find new solvable lattice models whose Boltzmann weights obey high genus algebraic relations generalizing the results of [1] for the case of other R-matrices.

This program for the case of the three state R-matrix of [8] which is related to the $U_q(sl(3))$ algebra at $q^{2N}=1$ has been partially realized in [9, 10].

In the present paper we extend the result of [9, 10]. We construct an n-by-$N^{(n-1)}$ state cyclic L-operator related with an n-state R-matrix of [8] and find explicitly the corresponding $N^{(n-1)}$-state R-matrix. This result is described below.

Consider an oriented square lattice $\mathscr{L}$ and its medial lattice $\mathscr{L}'$ (shown in Fig. 1 by solid and dashed lines, respectively). The oriented vertical (horizontal) lines of $\mathscr{L}'$ carry rapidity variables p, p' (q, q') in alternating order (note that the orientations of rapidity lines shown by open arrows alternate, too). The edges of