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An Integral Representation and Bounds on the Effective Diffusivity in Passive Advection by Laminar and Turbulent Flows

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Abstract. Precise necessary and sufficient conditions on the velocity statistics for mean field behavior in advection-diffusion by a steady incompressible velocity field are developed here. Under these conditions, a rigorous Stieltjes integral representation for effective diffusivity in turbulent transport is derived. This representation is valid for all Péclet numbers and provides a rigorous resummation of the divergent perturbation expansion in powers of the Péclet number. One consequence of this representation is that convergent upper and lower bounds on effective diffusivity for all Peclet numbers can be obtained utilizing a prescribed finite number of terms in the perturbation series. Explicit rigorous examples of steady incompressible velocity fields are constructed which have effective diffusivities realizing the simplest upper or lower bounds for all Péclet numbers. A nonlocal variational principle for effective diffusivity is developed along with applications to advection-diffusion by random arrays of vortices. A new class of rigorous examples is introduced. These examples have an explicit Stieltjes measure for the effective diffusivity; furthermore, the effective diffusivity behaves like $\kappa_0 (Pe)^{1/2}$ in the limit of large Péclet numbers where κ_0 is the molecular diffusivity. Formal analogies with the theory of composite materials are exploited systematically.

Introduction

We are interested in the passive advection of a scalar quantity by a random, statistically homogeneous, incompressible, velocity field. This important problem arises commonly in the study of turbulent diffusion in the atmosphere [1], in dispersion of tracers in Bénard convection rolls [2] and porous materials [3], as

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