Commun. Math. Phys. 138, 273–290 (1991)

Communications in Mathematical Physics


# Topological Couplings and Contact Terms in 2d Field Theory

**Jacques Distler**[1] **and Philip Nelson**[2]

[1] Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA
[2] Physics Department, University of Pennsylvania, Philadelphia, PA 19104 USA

Received October 12, 1990

**Abstract.** In string theory and in topological quantum field theory one encounters operators whose effect in correlation functions is simply to measure the topology of 2d spacetime. In particular these "dilaton"-type operators count the number of other operators via contact terms with the latter. While contact terms in general have a reputation for being convention-dependent, the ones considered here are well-defined by virtue of their simple geometrical meaning: they reflect the geometry of the stable-curve compactification. We give an unambiguous prescription for their evaluation which involves no analytic continuation in momenta.

## 1. Introduction

Usually the various $n$-point correlation functions of a quantum field theory are related to each other in complicated, indirect ways. One obtains such relations, for example, from unitarity of the scattering matrix. In certain limiting cases one obtains very simple results, for example the famous low-energy theorems relating amplitudes with and without a zero-momentum pion. String theory reflects such relations on scattering amplitudes via affiliated relations between an $N$-point amplitude and the *integral* of an $(N+1)$-point amplitude over the location of the last point. More generally, the latter integral can be regarded as the change of the $n$-point functions of some conformal field theory as we deform the theory slightly. The statement then becomes that certain deformations modify amplitudes in very simple ways.

The most famous low-energy theorem in string theory is the statement that the zero-momentum mode of the dilaton is the string *coupling constant*. Thus deforming any amplitude by this operator merely *multiplies* it by $(1+\varepsilon)^M$, where $\varepsilon$ is the strength of the perturbation and $M$ is the order in which the given $n$-point function enters in string perturbation theory. In other words, the $(N+1)$-point function involving a zero-momentum dilaton, integrated over 2d spacetime, must equal $M$