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$gl(\infty)$ and Geometric Quantization

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Abstract. An axiomatic approach to the approximation of infinite dimensional algebras is presented; examples illustrating the need for a rigorous treatment of this subject. Geometric quantization is employed to construct systematically $su(N)$ approximations of diffeomorphism algebras which first appeared in the theory of relativistic membranes.

1. Introduction

Over the past years several authors [1] have studied (and used) the approximation of diffeomorphism groups by $SU(N)$. They started from the observation made in the context of membrane theories [2, 3], that in a specific basis of $su(N)$ the corresponding structure constants converge to those of $\mathrm{diff}_A S^2$ (the Lie algebra of infinitesimal area preserving diffeomorphisms of the 2-sphere) in the limit $N \to \infty$. Later it was found [4–7] that the same holds for the Lie algebra of infinitesimal (nonconstant) diffeomorphisms of the 2-torus $\mathrm{diff}_A T^2$.

A naive identification of $\mathrm{diff}_A S^2$ and $\mathrm{diff}_A T^2$, however, with the well known $su_{(+)}(\infty)$ [8] would be false. Although the three algebras (or rather certain subalgebras) may all be approximated by $su(N)$ in the above sense, they are pairwise non-isomorphic. This we will show in Appendix A. Moreover, in [9] the members of an infinite family of algebras (including $\mathrm{diff}_A T^2$) have been proven to be pairwise non-isomorphic although all of them can be approximated by $su(N)$, $N \to \infty$.

This ambiguity clearly shows the need for an additional concept, and appears to be worthwhile studying without reference to membrane theory. Interesting questions arising from the subject are its relation to $\hbar \to 0$ limits of quantum theories on compact phase spaces and its role in the construction and classification of infinite dimensional Lie algebras.