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Perturbation Theory for Periodic Orbits in a Class of Infinite Dimensional Hamiltonian Systems

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Abstract. We consider a class of Hamiltonian systems describing an infinite array of coupled anharmonic oscillators, and we study the bifurcation of periodic orbits off the equilibrium point. The family of orbits we construct can be parametrized by their periods which belong to Cantor sets of large measure containing certain periods of the linearized problem as accumulation points. The infinitely many holes forming a dense set on which the existence of a periodic orbit cannot be proven originate from a dense set of resonances that are present in the system. We also have a result concerning the existence of solutions of arbitrarily large amplitude.

1. Introduction

1.1. The purpose of this article is to study periodic solutions of infinite-dimensional Hamiltonian systems describing anharmonic oscillators with random spring constants located on the sites of the ν-dimensional cubic lattice, $\mathbb{Z}^\nu$. The equations of motion are

$$\frac{d^2}{dt^2} u(x,t) + [(-\Delta + V(x))u]\,(x,t) + \lambda W(u)\,(x,t)u(x,t) = 0\,, \tag{1.1}$$

where u is a real-valued function, $u : \mathbb{Z}^n \times \mathbb{R} \to \mathbb{R}$ and Δ is the discrete laplacian defined by

$$(\Delta u)\,(x,t) = \sum_{|y-x|=1} (u(y,t) - u(x,t))\,. \tag{1.2}$$

The spring constants $V(x)$ are i.i.d. random variables taking only positive values, with a probability distribution of $V \equiv V(x)$ given e.g. by

(i) $$d\varrho(V) = N\theta(V)e^{-|V|^\alpha/\zeta}dV\,, \tag{1.3}$$

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