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On the Algebraic Structure of Link-Diagrams on a 2-Dimensional Surface*

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Abstract. Following and generalizing a paper by Turaev, we consider some algebraic structures on the set of (generalized) link-diagrams, meant simply as collections of immersed loops on a two dimensional surface, with the specification of an over/undercrossing prescription at each double point. This definition is general enough to be relevant not only to traditional knot-theory, but possibly to some statistical mechanical models. A coalgebra structure is introduced on some modules (over polynomial rings), generated by these diagrams. The compatibility of this coalgebra structure with the skein invariance and the invariance under Reidemeister moves is discussed. A Hopf algebra structure results only in some special cases, which are thoroughly examined. It is shown that a special choice of the ground (polynomial) ring over which the diagram module is defined, allows us to define link-invariants for links (in the ordinary sense) in $\Sigma \times [0,1]$, where Σ is a (closed or open) two-dimensional surface. These invariants generalize in a non-trivial way the Jones polynomials and the Homfly polynomials (at least when the last ones are computed for some special values of the variables). In a sequel paper the relation between the algebraic structures of link-diagrams, some special types of quantum groups and the quantum holonomy will be discussed.

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