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Gauge Theory in Witten's Approach to the Generation Problem

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Abstract. In Witten's topological theory of the generation problem, gauge groups are identified with the E_8 centraliser of the holonomy group of the internal manifold. Here we show that this amounts to interpreting gauge groups as generalised symmetry groups of the (internal) Levi-Civitá connection. We then give techniques for computing centralisers in exceptional groups, taking into account the fact that holonomy groups are frequently disconnected. These techniques allow us to deal with compact locally irreducible Ricci-flat Riemannian manifolds of all holonomy types and dimensions.

1. Introduction

In view of the recent experimental work on determining the number of light generations of fermions, it is timely to reconsider how the matter lies from a theoretical standpoint. How are we to understand the existence of the two apparently superfluous generations? Candelas et al. (1985) proposed that the answer lies in the deep structure of the Dirac equation, in the relationship between its solutions and the topology of (multi-dimensional) space-time. This profound and beautiful approach to the generation problem leads to the simple relation

$$\# = \tfrac{1}{2}|\chi + \tau|,$$

where $\#$ is the number of generations, and χ and τ are respectively the Euler characteristic and the signature of the internal manifold (Green et al., 1987). These ideas are almost invariably associated with 10-dimensional string theory; but this is not really necessary, as the account given by Green et al. (1987) makes quite clear. Indeed, Witten (1985) sketched the main ideas well before the advent of the heterotic string. In view of the importance of the problem, it seems to us that the approach of Witten (1985) and Candelas et al. (1985) is worthy of study in its own right, independent of its association with string theory. In this work, then, we shall study the foundations of this approach, regarding it mainly as a contribution