

# Monotonicity Properties of the Family of Trapezoidal Maps

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**Abstract.** Trapezoidal maps which are everywhere expanding out of their plateau form a three parameter family  $T$ , up to affine changes of coordinates. We show that splitting  $T$  according to the various possible dynamical “behaviors” (we make this word precise in the process), yields a codimension one foliation. Some consequences of our result in terms of the monotonicity along simple one parameter families in  $T$  are then drawn. All together, aperiodic behavior is rare both from the topological and the measure theoretical point of view in  $T$ .

## I. Introduction and Statement of the Results

In this paper, we consider the set  $T$  of “trapezoidal maps” (i.e., maps whose graph is trapezoidal) on the unit interval  $I = [0, 1]$ , which absolute values of the slopes on both sides of the plateau greater than one, and which send both endpoints of  $I$  to 0. These maps have been the object of many publications (see Sect. III), and it turns out that most of the questions about the evolution of the topological dynamics in one parameter families of such maps (see Theorem 1), and a related measure theoretical question (see Theorem 2) can be easily deduced from kneading theory [4, 8, 9] and the following well known:

*Main Fact.* For a map in  $T$ , the Lebesgue measure of the union of all inverse images of the plateau  $\mathcal{P}$  is one, i.e.,  $\lambda\left(\bigcup_{n=0}^{\infty} f^{-n}(\mathcal{P})\right) = 1$ .

For completeness a proof of this statement will be provided in the Appendix.

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