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Lie Superalgebraic Approach to Super Toda Lattice and Generalized Super KdV Equations*

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Abstract. We propose a super Lax type equation based on a certain class of Lie superalgebra as a supersymmetric extension of generalized (modified) KdV hierarchy. We are able to construct an infinite set of conservation laws and the consistent time evolution generators for generalized modified super KdV equations. The first few of the conserved currents, the (modified) super KdV equation and the super Miura transformation are worked out explicitly in the case of twisted affine Lie superalgebra $OSp(2|2)^{(2)}$.

1. Introduction

Of integrable nonlinear systems generalized Korteweg-de Vries (KdV) equations and Toda lattice equations are particularly interesting classes in connection with conformal field theories. The Virasoro algebra can be extended to W_n algebra [1] by incorporating conserved currents of higher spin. The W_n algebra is known to arise from the Hamiltonian structure of the generalized KdV equation [2, 3]. It has recently been shown [4] that perturbation of conformal field theories by certain types of interaction is described effectively by affine Toda lattice theories [5]. Some time ago Drinfeld and Sokolov developed a Lie algebraic method [2] to derive generalized KdV equations and relate them to affine Toda lattice equations. This method is also related to the method of coadjoint orbit and Hamiltonian reduction of current algebras.

In this paper we extend the Lie algebraic method of Drinfeld and Sokolov to the sypersymmetric case and develop a Lie superalgebraic method for generalized super KdV and super Toda lattice equations. Drinfeld and Sokolov introduced a

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