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Yang-Baxterization of Braid Group Representations

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Abstract. For a given braid group representation (BGR), a process of the Yang-Baxterization is formulated to generate solutions of the Yang-Baxter equation (YBE). When a BGR admits the Birman-Wenzl (BW) algebraic structure, this process can be explicitly passed through and two types of trigonometric solutions of YBE are generated from such a BGR. These two solutions have the essential difference to each other and both of them preserve the crossing symmetry property if the given BGR has. By taking certain reduction on the BW algebra, the rational solution is also generated. A practical condition to judge whether a BGR satisfies the BW algebra is given, from which one finds that not only the familiar BGRs of [5,7,9], but also some new ones obtained recently in [12] have the BW structure. Thus they can be explicitly Yang-Baxterized to solutions of the YBE.

1. Introduction

It is known, nowadays, that the Yang-Baxter equation (YBE) plays a central role in the study of integrable models in statistical mechanics and quantum field theory, and is also closely related to some other fields, such as the quantum group, knot theory and conformal field theory etc., in both mathematics and physics [1–4].

Based on the theory of quantum group, or the q-analogues of the universal enveloping Lie algebras, Jimbo [5] constructed a family of trigonometric solutions of the YBE associated with the fundamental representations of the Kac-Moody algebras, and they possess the classical limits of Belavin and Drinfeld [6]. By letting the spectral parameter be zero (or infinity in a different choice) on the other hand, these solutions give rise to the braid group representations (BGRs)

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