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Renormalization Group Analysis of a Simple Hierarchical Fermion Model*

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Abstract. A simple hierarchical fermion model is constructed which gives rise to an exact renormalization transformation in a 2-dimensional parameter space. The behaviour of this transformation is studied. It has two hyperbolic fixed points for which the existence of a *global* critical line is proven. The asymptotic behaviour of the transformation is used to prove the existence of the thermodynamic limit in a certain domain in parameter space. Also the existence of a continuum limit for these theories is investigated using information about the asymptotic renormalization behaviour. It turns out that the "trivial" fixed point gives rise to a two-parameter family of continuum limits corresponding to that part of parameter space where the renormalization trajectories originate at this fixed point. Although the model is not very realistic it serves as a simple example of the application of the renormalization group to proving the existence of the thermodynamic limit and the continuum limit of lattice models. Moreover, it illustrates possible complications that can arise in global renormalization group behaviour, and that might also be present in other models where no global analysis of the renormalization transformation has yet been achieved.

1. Introduction

Hierarchical models were introduced by Dyson [8] before Wilson [19] formulated his theory of the renormalization group. It was Baker [1] who pointed out the simple renormalization group structure of the model. Actually, Baker's model is different from Dyson's in that it has continuous spins instead of Ising spins. The first mathematical investigation of hierarchical models was carried out by Bleher

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