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The Fixed Boundary Value Problems for the Equations of Ideal Magneto-Hydrodynamics with a Perfectly Conducting Wall Condition

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Abstract. The equations of ideal Magneto-Hydrodynamics are investigated concerning initial boundary value problems with a perfectly conducting wall condition. The local in time solution is proved to exist uniquely, provided that the normal component of the initial magnetic field vanishes everywhere or nowhere on the boundary.

1. Introduction

The equations of ideal Magneto-Hydrodynamics (ideal MHD) are the model which describes the macroscopic motion of an electrically conducting fluid. Here "ideal" means the model to be free from the effect of viscosity and electrical resistivity. In this paper we study initial boundary value problems for the ideal MHD with a perfectly (electrically) conducting wall condition.

Although there are several studies of these problems relevant to the plasma confinement from physical and engineering viewpoints (cf. [6]), any mathematical exploration into these problems, as far as we know, has not been found. Even the boundary conditions themselves, which are not only mathematically proper but also fully consistent with the physical situation, have not been well investigated. Therefore we first investigate and propose such adequate boundary conditions to a perfectly conducting wall. Then we show local in time existence of a unique classical solution to two special cases of these conditions. Now we state our problems more precisely. The problems we will treat are the equations of ideal MHD,

$$\varrho_p(\partial_t+(u\cdot\nabla))p+\varrho\nabla\cdot u=0\,, \tag{1.1$_\mathrm{a}$}$$

$$\varrho(\partial_t+(u\cdot\nabla))u+\nabla p+\mu H\times(\nabla\times H)=0\,, \tag{1.1$_\mathrm{b}$}$$

$$\partial_t H-\nabla\times(u\times H)=0\,,\quad \text{in}\quad [0,T]\times\Omega\,, \tag{1.1$_\mathrm{c}$}$$

$$(\partial_t+(u\cdot\nabla))S=0\,, \tag{1.1$_\mathrm{d}$}$$

$$\nabla\cdot H=0\,, \tag{1.1$_\mathrm{e}$}$$