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Exponential Decay of Green's Functions for a Class of Long Range Hamiltonians

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Abstract. We consider a class of long range Hamiltonians with diagonal disorder on $l^2(\mathbf{Z})$. For any *ergodic* potential V with non-empty essential range, we prove the exponential decay of the Green's functions for energies in the essential range. If V is independent identically distributed, we obtain the exponential decay of the Green's functions for all coupling constant $\lambda > 0$. Moreover the Hamiltonian has only pure point spectrum.

1. Introduction

We consider a long range Hamiltonian with diagonal disorder. Let (Ω, P) be a probability space, $T^j (j \in \mathbf{Z})$ a one parameter group of ergodic measure preserving transformations acting on Ω; f a measurable real-valued function on Ω, such that the distribution P^f of f has no point component: $P\{\omega \mid f(\omega) = E\} = 0$ for any E. Let the corresponding *ergodic* potential $V^\omega(j) = f(T^j\omega)$; we define H^ω on $l^2(\mathbf{Z})$ by:

$$H^\omega = \tilde{\Delta} + \lambda V^\omega, \tag{1.1}$$

where $\tilde{\Delta}$ is a long-range finite difference Laplacian:

$$\tilde{\Delta}(m,n) = \frac{\alpha^{-|m-n|+1}}{1-\alpha^2} \quad (\alpha > 1),$$

($1/(1-\alpha^2)$ is just a normalizing factor), and λ is the coupling constant. (We will write V, H, instead of V^ω, H^ω for convenience.)

Clearly $\tilde{\Delta}$ is very different from the usual finite difference Laplacian Δ: $\Delta(m,n) = \delta(|m-n|-1)$, and is perhaps physically more realistic.

We are interested in the asymptotic behavior of the Green's functions for H. We show in the next section that H can be written as

$$H = \frac{1}{\Delta - (\alpha + \alpha^{-1})} + \lambda V. \tag{1.1a}$$