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Zero-Time One-Particle States in Quantum Field Theory

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Abstract. In the framework of the weakly-coupled $P(\varphi)_2$-models we construct perturbation approximations of vectors of a dense set of the state space, especially vectors of the one-particle state subspace, by polynomials of zero-time fields acting on the vacuum state, with rigorous control of the remainders.

Introduction

Motivation. The particle structure of a Quantum Field Theory model is generally deduced from the analyticity properties of the Green distributions, obtained from the study of the Bethe–Salpeter equation. This gives precise information about the spectrum of M, the mass operator (see the references in [2], to which we must add now [1]). Another method, not completely independent, is the *variational perturbation method*, intially proposed by Glimm, Jaffe and Spencer [8], and studied in [2] for same $P(\varphi)_2$-models. For all $f \in L^1 \cap L^2(\mathbb{R}^2)$, $f \neq 0$, a vector $\Psi(f)$ is constructed, which satisfies the following conditions:

1. it is a linear combination of zero-time fields acting on the vacuum, 2. it is orthogonal to the vacuum and to the one-particle states, 3. it lies in the domain of M, and 4. it verifies the following formula:

$$\frac{(\Psi(f); M^2 \Psi(f))}{(\Psi(f); \Psi(f))} = (2m)^2 + 4\frac{\lambda^2}{m}\frac{\langle f; H_{\mathrm{rel}}^{\mathrm{NR}} f\rangle}{\langle f; f\rangle} + O(\lambda^{5/2})$$

for λ, the coupling constant, sufficiently small. Here $(\cdot;\cdot)$ is the state-space scalar product, m is the one-particle physical mass, $\langle\cdot;\cdot\rangle$ is the $L^2(\mathbb{R}^2)$ scalar product and $H_{\mathrm{rel}}^{\mathrm{NR}}$ is the relative Hamiltonian of the Non-Relativistic limit, written for $\lambda = m^2$. A careful study of the remainder, which is $O(\lambda^{5/2})$, has been made. The above formula has been established for all $P(\varphi)_2$-models with even interaction polynomial P having a non-zero fourth degree term. For a discussion on the conclusions that can be drawn apropos of the spectrum of M and its eigenvectors, see [2].