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Localization in the Ground State of the Ising Model with a Random Transverse Field

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Abstract. We study the zero-temperature behavior of the Ising model in the presence of a random transverse field. The Hamiltonian is given by

$$H = -J \sum_{\langle x,y \rangle} \sigma_3(x)\sigma_3(y) - \sum_x h(x)\sigma_1(x),$$

where $J > 0$, $x, y \in \mathbf{Z}^d$, σ_1, σ_3 are the usual Pauli spin $\frac{1}{2}$ matrices, and $\mathbf{h} = \{h(x), x \in \mathbf{Z}^d\}$ are independent identically distributed random variables. We consider the ground state correlation function $\langle \sigma_3(x)\sigma_3(y) \rangle$ and prove:

1. Let d be arbitrary. For any $m > 0$ and J sufficiently small we have, for almost every choice of the random transverse field $\mathbf{h}$ and every $x \in \mathbf{Z}^d$, that

$$\langle \sigma_3(x)\sigma_3(y) \rangle \leqq C_{x,\mathbf{h}} e^{-m|x-y|}$$

for all $y \in \mathbf{Z}^d$ with $C_{x,\mathbf{h}} < \infty$.

2. Let $d \geqq 2$. If J is sufficiently large, then, for almost every choice of the random transverse field $\mathbf{h}$, the model exhibits long range order, i.e.,

$$\overline{\lim_{|y| \to \infty}} \langle \sigma_3(x)\sigma_3(y) \rangle > 0$$

for any $x \in \mathbf{Z}^d$.

1. Introduction

Quantum spin systems with random parameters have been introduced to study the effects of impurities in several physical systems (see for example, Halperin, Lee

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