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Potentials on the Two-Torus for Which the Hamiltonian Flow is Ergodic

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Abstract. We consider the motion of a test particle in a compound central potential field on a two-dimensional torus. We discuss three different classes of potentials (attracting, repelling, and mixed) that lead to Hamiltonian systems which have positive Lyapunov exponent almost everywhere and are ergodic. Included among the mixed potentials are smooth potentials without singularities.

1. Introduction

Do gas molecules interacting in a box behave stochastically? Boltzmann's ergodic hypothesis, rephrased in modern language, asserts the affirmative. In the nineteen forties, the Russian physicist Krylov [Kr] studied the case of a gas of hard spheres. His calculations indicated that collisions between the spheres would lead to an exponential divergence of trajectories. Hopf [H] had recently shown that geodesic flow on surfaces of negative curvature was ergodic precisely because of such exponential instability of trajectories. Krylov argued that the hard sphere gas should therefore also behave stochastically.

In the early 1960s, Sinai [Si1] continued the work of Krylov. He translated the problem of two hard spheres into a billiard system consisting of one particle moving on a two dimensional torus T^2 with circular obstacles (scatterers). Sinai [Si2] showed that this billiard system was ergodic. Furthermore, recent work has proven ergodicity of systems of three and four balls [SC, KSS1, KSS2].

We will study a related systems, also discussed by Krylov, obtained by replacing the circular scatters on the billiard table by symmetric potentials of finite range. One then examines the motion of a point particle in the potential field. The resulting

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