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# Ergodicity of Probabilistic Cellular Automata: A Constructive Criterion

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**Abstract.** We give a sequence of criteria (of increasing complexity) for the exponential ergodicity of discrete time interacting particle systems. Each criterion involves estimating the dependence on initial conditions of the process on finite space-time volumes. It generalizes and improves the existing single site condition and is the analog of the Dobrushin–Shlosman $C_V$ condition in equilibrium statistical mechanics. Our "dynamic" criterion may also be used to prove the uniqueness of Gibbs state in situations where the $C_V$ condition fails. As a converse we prove that if there is a certain form of convergence to the stationary measure faster than $n^{-d}$, where $n$ is the time and $d$ is the dimension of the lattice, then our condition holds for some space–time volumes and hence the convergence must be exponentially fast.

## 1. Introduction

It is well known that finite state space Markov chains have a unique stationary measure whenever there is a strictly positive probability to go from one state to any other in a finite time not depending on the pair of states (irreducible and aperiodic Markov chains), [F, Ge]. The situation is very different for the so-called Probabilistic Cellular Automata (PCA) – Markov chains, for which an infinite collection of cells, each assigned to the vertex of some lattice and taking on a finite number of positions, evolve under a discrete time stochastic dynamics. There exist in fact many examples of PCA with strictly positive transition probabilities which are not ergodic. Even so, it is in general rather easy to give sufficient conditions which ensure the ergodicity of a PCA, see for example [V, LMS, KMPSTV, Fe, S].

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