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Breaking of Periodicity at Positive Temperatures

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Abstract. We discuss a classical lattice gas model without periodic or quasiperiodic ground states. The only ground state configurations of our model are nonperiodic Thue-Morse sequences. We show that low temperature phases of such models can be ordered. In fact, we prove the existence of an ordered (nonmixing) low temperature translation invariant equilibrium state which has nonperiodic Gibbs states in its extremal decomposition.

A number of classical lattice gas models without periodic ground state configurations have been found recently. So far, they were of quasiperiodic (in the sense described below) or incommensurate character. In the first category we have two-dimensional nearest neighbor interactions studied recently by Radin and Miękisz [1–9] (for related ideas in a somewhat different setup see [10, 11]). The models are based on results on nonperiodic tilings of the plane with a finite family of square-like tiles [12–15]. The ground state configurations of these models are nonperiodic structures minimizing the energy of all bonds simultaneously (interactions are not competing). They are quasiperiodic in the following sense: if a certain fraction of particles is ignored the rest of a ground state configuration is periodic; the smaller the fraction, the larger the period. Incommensurate ground state configurations, on the other hand, arise as a result of competition between interactions with incommensurate length scales [16, 17].

Recently, an example was given of a one-dimensional exponentially decaying Ising-type non-competing interaction with ground state configurations which are neither quasiperiodic nor incommensurate [18]. The unique ground state (the unique translation invariant probability measure supported by ground state configurations) of this model has two unusual properties: it has perfect fractal symmetry in the sense that the structure of the ground state is invariant under certain scale changes and the translation (shift) operator has some continuous spectrum. The ground state, as in the previous models, is not mixing. We say that it has long range order in the sense that distant regions are correlated.

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