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Quantum R Matrices Related to the Spin Representations of B_n and D_n

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Abstract. We present the explicit form of the trigonometric R matrices related to the spin representations of the simple Lie algebras $X_n = B_n, D_n$. We conjecture that one dimensional configuration sums of the corresponding vertex models in the face formulation are the string functions of $X_n^{(1)}$ modules.

1. Introduction

The importance of quantum R matrices has been recognized widely these days because of its deep relationship with quantum groups, q-analysis, operator algebras, like invariants, conformal field theories, statistical mechanical models, etc. In constructing trigonometric R matrices, the quantized universal enveloping algebra $U_q\mathfrak{g}$ plays a significant role. In [1] V. G. Drinfeld constructed a "universal R matrix" $\mathscr{R} \in U_q\mathfrak{g} \otimes U_q\mathfrak{g}$. This, in principle, enables us to write down the form of the R matrix corresponding to an arbitrary pair of a nontwisted affine Lie algebra $\hat{\mathfrak{g}}$ and an irreducible representation π of $\mathfrak{g}$. From the statistical mechanical point of view, each R matrix defines a solvable vertex model on the two dimensional square lattice. In order to carry out its analysis, we have to deal with the explicit form of the R matrix. So far, such explicit expressions have been obtained in the case of $\hat{\mathfrak{g}} = A_1^{(1)}, \pi =$ an arbitrary representation [2, 3] and in the case of $\hat{\mathfrak{g}} = A_n^{(1)}, B_n^{(1)}, C_n^{(1)}, D_n^{(1)}, \pi =$ the vector representation [4, 5]. Very recently, an exceptional case $G_2^{(1)}$ is also treated in [6].

In [3] a method was initiated to construct the R matrices related to higher representations from the one related to a basic representation. This method is called the "fusion procedure." In the case of $A_n^{(1)}$, the key R matrix is the one corresponding to the vector representation. I. V. Cherednik worked out the fusion procedure in the elliptic parametrization [7]. When we consider the cases of $B_n^{(1)}$ and $D_n^{(1)}$, the R matrices corresponding to the spin representations are necessary for the fusion procedure. The purpose of this article is to give a concise form of the trigonometric R matrices related to the spin representations of B_n and D_n.