

Super Loop Groups, Hamiltonian Actions and Super Virasoro Algebras*

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Abstract. The quotient $\widetilde{LG}/G$ of a super loop group $\widetilde{LG}$ by the subgroup of constant loops is given a supersymplectic structure and identified through a moment map embedding $\tilde{J}^{L^{\wedge}}: \widetilde{LG}/G \rightarrow \widetilde{Lg}^{\wedge*}$ with a coadjoint orbit of the centrally extended super loop algebra $\widetilde{Lg}^{\wedge}$. The algebra $\widetilde{\text{diff}}^c S^1$ of super-conformal vector fields on the circle is shown to have a natural representation as Hamiltonian vector fields on $\widetilde{LG}/G$ generated by an equivariant moment map $\tilde{J}: \widetilde{LG}/G \rightarrow \widetilde{\text{diff}}^c S^{1*}$. This map is obtained by composition of $\tilde{J}^{L^{\wedge}}$ with a super Poisson map $\tilde{J}^{\mathcal{J}}: \widetilde{Lg}^{\wedge*} \rightarrow \widetilde{\text{diff}}^c S^{1*}$ defining a supersymmetric extension of the classical Sugawara formula. Upon quantization, this yields the corresponding formula of Kac and Todorov on unitary highest weight representations. For any homomorphism $\rho: \mathfrak{u}(1) \rightarrow G$, an associated “twisted” moment map $\tilde{J}^{\rho}: \widetilde{LG}/G \rightarrow \widetilde{\text{diff}}_{\mathbb{C}} S^{1*}$ is also derived, generating a super Poisson bracket realization of a super Virasoro subalgebra $\widetilde{\text{Vir}}$ of the semi-direct sum $\widetilde{\text{diff}}_{\mathbb{C}} S^1 \ltimes \widetilde{Lg}^{\wedge}$. The corresponding super Poisson map $\tilde{J}^{\mathcal{J}\rho}: \widetilde{Lg}^{\wedge*} \rightarrow \widetilde{\text{Vir}}^*$ is interpreted as a nonabelian generalization of the super Miura map and applied to two super KdV hierarchies to derive corresponding integrable generalized super MKdV hierarchies in $\widetilde{Lg}^{\wedge*}$.

1. Introduction

In the Hamiltonian framework, the classical Sugawara formula [S] for a $1+1$ dimensional field theory may be viewed as a Poisson map $J^{\mathcal{J}}: Lg^{\wedge*} \rightarrow \text{diff} S^{1*}$ from the dual of the centrally extended loop algebra $Lg^{\wedge}$ to the dual of the algebra $\text{diff} S^1$ of vector fields on the circle. The Poisson space $Lg^{\wedge*}$ enters as a “universal” phase space into which conformally invariant models may be mapped via their currents. The latter are interpreted in the Hamiltonian framework as moment maps generating the action of the loop group LG in terms of Hamiltonian flows. The

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