Commun. Math. Phys. 132, 131–138 (1990)

Communications in
Mathematical
Physics


# On the Exchange Matrix for WZNW Model

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*Dedicated to Res Jost and Arthur Wightman*

**Abstract.** The connection between the exchange algebra in the $SU(2)$ Wess-Zumino Novikov-Witten model and the quantum group $SU(2)$ is discussed. It is shown that on the quasiclassical level this connection has the simple interpretation in terms of the Lie-Poisson action of $SU(2)$ on the chiral components of the fields in the WZNW model.

Nowadays a new tendency proclaims itself in $2d$ conformal field theory. In contrast to the pure bootstrap program of the original proposal by Belavin, Polyakov and Zamolodchikov [1] it is based on more traditional methods of quantum field theory. We see the return of the lagrangian formulation with the beautiful geometric interpretation of corresponding action (see i.e. [2]), use of the functional methods [3], etc. In view of the exact solvability of CFT, it is natural to invoke the approach of quantum integrable models [4]. In fact, it was extensively used in the beginning of 80-ties in connection with the Liouville model [5–7]. Moreover, the quantum group attributes already appeared in these papers. Now, when the quantum group methods are coming to fashion it is only reasonable to return to this approach and take off the mystery from the relations between conformal field theory and quantum groups.

A recent preprint by Gervais [8] on the Liouville model is one of the first steps in this direction. My note contains some results obtained independently. I have taken as a representative example the $SU(2)$ Wess-Zumino-Novikov-Witten model rather than Liouville (which is a reduction of noncompact $sl(2, \mathbf{R})$ WZNW) because the peculiarity of the elliptic monodromy is clearer there.

I shall work in the hyperbolic space time with the simplest topology of a cylinder,

$$0 \leqq x \leqq 2\pi; \qquad -\infty < t < \infty .$$

The classical field variable $g(x, t)$ is a $2 \times 2$ unitary unimodular matrix, periodic in $x$

$$g(x+2\pi, t) = g(x, t) .$$