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The Computation of Characteristic Classes of Lattice Gauge Fields

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Abstract. A $GL(p, \mathbf{C})$-valued lattice gauge field $\mathbf{u}$ on a simplicial complex determines a principal $GL(p, \mathbf{C})$-bundle ξ if the plaquette products are sufficiently small with respect to the maximum distortion coefficient of the transporters. A representative cocycle c_q for the q^{th} Chern class of ξ can be computed on each $2q$-simplex σ by taking $c_q(\sigma)$ to be the intersection number of a certain singular $2q$-cube M_σ with a Schubert-type variety Σ_q in the space of all $p \times p$ matrices. This reduces to the solution of polynomial equations with coefficients coming from $\mathbf{u}$ and thus avoids numerical integration or cooling-type procedures. An application of this method is suggested for the computation of the topological charge of an $SU(3)$-valued lattice gauge field on a 4-complex.

Introduction

This work grew out of our earlier research in the topology of lattice gauge fields [23, 24], which in turn was inspired by the work of Martin Lüscher [17]. There we gave algorithms for the computation of the characteristic numbers of $U(1)$ and $SU(2)$-valued lattice gauge fields on triangulated 2 and 4-dimensional manifolds. Here we examine the problem of computing characteristic classes of lattice gauge fields with values in $GL(p, \mathbf{C})$, for arbitrary p, defined on simplicial complexes of arbitrary dimension. In particular this work could be used as the basis of a new algorithm for the evaluation of the topological charge of an $SU(3)$-valued lattice gauge field on a 4-dimensional complex, a problem that has recently been examined from a wide variety of angles [3, 7, 8, 10, 13, 14, 15, 22], some of them reviewed in [12] along with the physical context of the problem. In our $SU(2)$ work we were able to exploit the extremely simple geometry of the group: since geodesics on a 3-sphere are its intersections with 2-planes in 4-space, most questions about relative

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