Quantum Geometry of Loops and the Exact Solubility of Non-Abelian Gauge Chern-Simons Theory – II

M. Awada*

Physics Department, Imperial College, London SW7, UK

Abstract. We quantize non-abelian Chern-Simons gauge theory in three dimensions in the presence of Wilson lines. We determine the theory dynamically in terms of the geometry of loops and show that it is exactly soluble. Remarkably the quantum loop equations are linear for S^3 and they possess a class of solutions, among which is a non-critical Fermi string theory. Using these solutions we determine various important identities relevant to knot theory discovered recently by E. Witten, in particular, we show that the loop equation yields precisely the full exact skein relation of knot theory. As a byproduct we show that the partition function of an unknotted Wilson loop on S^3 is nothing but the character of $SU(2)$ in which the rotations are $SU(N)$-valued fractional angles. Furthermore, we generalize our solutions to the case where the manifold M_3 is oriented, closed, and non-simply connected with $H_1(M_3) = 0$ (a homology 3-sphere).

1. Introduction

It has been long conjectured that closed strings play the role of elementary excitations in the confining phase of a gauge theory [1, 2]. This has led to various attempts to formulate Q.C.D. as a chiral theory in loop space [3, 4]. The strong similarity between the field equations of chiral fields in $d = 2$ space-time dimensions and the Yang-Mills field equations in loop space suggest that there exists an exact solution to the gluon dynamics. In fact, it has been suggested that in the $N = \infty$ limit, Q.C.D. is equivalent to a Fermionic string theory [5]. Unfortunately this scheme did not go very far due to various problems in the renormalization theory of fields on loop space. Nevertheless, the idea itself is very attractive and has potential applications in other types of non-abelian gauge theories. In this paper, we will formulate the Chern-Simons (CS) non-abelian gauge theory in three space-time dimensions in terms of quantum geometry of loops. We will show that the theory is exactly soluble in loop space, and is equivalent to a Fermionic string theory. Our approach will be an alternative new

* Address after 1ˢᵗ October 1989: Physics Department, University of Florida, Gainsville, FL 32611, USA