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Linearly Stable Orbits in 3 Dimensional Billiards*

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Abstract. We construct linearly stable periodic orbits in a class of billiard systems in 3 dimensional domains with boundaries containing semispheres arbitrarily far apart. It shows that the results about planar billiard systems in domains with convex boundaries which have nonvanishing Lyapunov exponents cannot be easily extended to 3 dimensions.

0. Introduction

Since the work of Sinai [S] we know that dispersing billiards (billiard systems in domains with concave boundaries) have strong mixing properties in all of the phase space. Conceptually such systems are close to geodesic flows on manifolds of nonpositive sectional curvature. In particular the dimension of the domain does not affect the basic features of the dynamics [C–S]. Bunimovich [B1] discovered that convex pieces of the boundary may produce the same scattering effect in planar billiards. He constructed examples of convex planar domains built of arcs of circles and straight lines (the stadium) in which billiard systems have the hyperbolic behavior. In [W1] the rigid requirement of constant curvature on the convex pieces of the boundary was replaced by the condition that $r''(s) \leqq 0$, where $r(s)$ is the radius of curvature as a function of arc length. Subsequently Markarian [M] and Donnay [D] showed that other conditions may suffice. It was already clear in [W1] that putting two convex pieces of the boundary sufficiently far apart makes the orbits that go back and forth between the two completely unstable (the difficulty there is to include the "glancing" pieces of the orbit: many consecutive reflections at large angles in one convex piece of the boundary). This idea is further supported by the work of Donnay, who shows that basically arbitrary sufficiently small perturbations of the stadium have nonzero Lyapunov exponents. (Bunimovich [B2] made some vague claims along these lines before but so far he has not published the details.) The natural question arises: do these planar (locally) convex domains have higher dimensional counterparts? More specifically we can ask if

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