Commun. Math. Phys. 128, 437–495 (1990)

Communications in Mathematical Physics

Hyperbolicity and Invariant Measures for General C^2 Interval Maps Satisfying the Misiurewicz Condition

Sebastian van Strien

Faculteit der Technische Wiskunde und Informatica, Technische Universiteit Delft, Julianalaan 132, NL-2628 BL Delft, The Nethérlands

Abstract. In this paper we will show that piecewise C^2 mappings f on $[0,1]$ or S^1 satisfying the so-called Misiurewicz conditions are globally expanding (in the sense defined below) and have absolute continuous invariant probability measures of positive entropy. We do not need assumptions on the Schwarzian derivative of these maps. Instead we need the conditions that f is piecewise C^2, that all critical points of f are "non-flat," and that f has no periodic attractors. Our proof gives an algorithm to verify this last condition. Our result implies the result of Misiurewicz in [Mi] (where only maps with negative Schwarzian derivatives are considered). Moreover, as a byproduct, the present paper implies (and simplifies the proof of) the results of Mañé in [Ma], who considers general C^2 maps (without conditions on the Schwarzian derivative), and restricts attention to points whose forward orbit stay away from the critical points. One of the main complications will be that in this paper we want to prove the existence of invariant measures and therefore have to consider points whose iterations come arbitrarily close to critical points. Misiurewicz deals with this problem using an assumption on the Schwarzian derivative of the map. This assumption implies very good control of the non-linearity of f^n, even for high n. In order to deal with this non-linearity, without an assumption on the Schwarzian derivative, we use the tools of [M.S.]. It will turn out that the estimates we obtain are so precise that the existence of invariant measures can be proved in a very simple way (in some sense much simpler than in [Mi]). The existence of these invariant measures under such general conditions was already conjectured a decade ago.

Introduction

There are a large number of papers on iterations of piecewise smooth one-dimensional mappings $f: M \to M$, where $M = [0,1]$ or S^1. Initially all metric results for these maps assumed that f is piecewise expanding, see for example [La, Y.]. Later the condition that f needs to be expanding was somewhat relaxed. This was done by considering expanding maps which are induced from special maps, see