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## The Polyakov Path Integral over Bordered Surfaces (The Open String Amplitudes)

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Abstract. We use the Feynman functional quantization scheme adapted to the gauge theories with reparametrization invariance to the functional covariant first quantization of the open bosonic BDHP string in a position representation. The consistent functional integral representation of the open string propagator is derived and evaluated. This result is used as a starting point for two kinds of constructions of the off-shell multiloop open string amplitudes. The general idea of the presented approach is to consider the off-shell amplitudes as functionals on the space  $\mathscr{C}$  of contours endowed with an intrinsic metric or on the space  $\mathscr{C}/\mathbb{R}_+$ .

## 1. Introduction

The Polyakov path integral [1] over bordered surfaces was first considered in the context of dual models beyond the critical dimensions [2]. At the same time its application to the phenomenological model for the Wilson loop was discussed in a more general framework by O. Alvarez [3]. In the current renewal of interest in quantum string theory it has become clear that the Polyakov path integral provides an efficient, manifestly covariant description of string theory at the critical dimension [4–6]. The basic objects of the modern S-matrix formulation are the on-shell multiloop amplitudes defined by means of the Polyakov functional integral over surfaces with prescribed topology and with vertex functionals corresponding to the ingoing and outgoing on-shell particle states [7]. The advantage of this approach is that the Polvakov functional measure is reduced via the Faddeev-Popov procedure to the uniquely determined measure on moduli space [3-6]. As a result, on-shell amplitudes can be written as finite dimensional integrals over moduli space of expressions made up of the known functions [8]. In the past few years, important progress has been made in the investigation of these amplitudes, especially in the case of the closed string [4–9]. Within the S-matrix formulation, the sum over bordered surfaces appears in the functional integral representation of the on-shell open string amplitudes [10].