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Harish-Chandra Isomorphisms for Quantum Algebras

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Abstract. The center of the quantum algebra is studied. Especially an analogue of the Harish-Chandra isomorphism is established.

1. Introduction

In the study of the quantum Yang–Baxter equation, Drinfel'd [3] and Jimbo [6] found a certain Hopf algebra, which is a quantization of the enveloping algebra of a symmetrizable Kac–Moody Lie algebra (The $\mathfrak{sl}_2$ case is due to Kulish-Reshetikhin and Sklyanin). The purpose of this paper is to investigate the structure of the center of this *quantum algebra* associated to a finite dimensional semisimple Lie algebra. Our main result is Theorem 2 below giving an analogue of the Harish-Chandra isomorphism ([5]).

Let $A = (a_{ij})_{1 \leqq i,j \leqq l}$ be a symmetrizable generalized Cartan matrix. This means that A is a matrix of integers such that $a_{ii} = 2$, $a_{ij} \leqq 0$ for $i \neq j$ and there exist positive integers $d_1, \ldots, d_l$ satisfying $d_i a_{ij} = d_j a_{ji}$. We fix such $d_1, \ldots, d_l$. Let k be a field of characteristic zero. Choose a finite-dimensional k-vector space $\mathfrak{t}_0$ and elements $\alpha_1, \ldots, \alpha_l \in \mathfrak{t}_0^*, t_1, \ldots, t_l \in \mathfrak{t}_0$ satisfying the following conditions:

(a) $\{\alpha_1, \ldots, \alpha_l\}$ is linearly independent,
(b) $\{t_1, \ldots, t_l\}$ is linearly independent,
(c) $\alpha_i(t_j) = d_i a_{ij}$ $(i, j = 1, \ldots, l)$.

The Kac–Moody Lie algebra $\mathfrak{g}$ (see [8]) associated to A is the Lie algebra over k, generated by the k-vector space $\mathfrak{t}_0$ and the elements $e_1, \ldots, e_l, f_1, \ldots, f_l$ with the following fundamental relations:

$$[t, t'] = 0 \quad (t, t' \in \mathfrak{t}_0), \tag{1.1}$$

$$[t, e_i] = \alpha_i(t) e_i \quad (t \in \mathfrak{t}_0, i = 1, \ldots, l), \tag{1.2}$$

$$[t, f_i] = -\alpha_i(t) f_i \quad (t \in \mathfrak{t}_0, i = 1, \ldots, l), \tag{1.3}$$

$$[e_i, f_j] = \delta_{i,j} t_i / d_i \quad (i, j = 1, \ldots, l), \tag{1.4}$$