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A Phase Cell Approach to Yang–Mills Theory V. Analysis of a Chunk*

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Abstract. In the present formalism the Yang–Mills field is constructed as a "non-linear sum" of excitations, small field excitations, the modes, and large field excitations, the chunks. The chunk excitations, herein studied, are each described by a finite number of group element variables. The continuum field associated to the excitation in general has point gauge singularities (arising from the non-trivial $\pi_3(G)$). We find estimates for plaquette assignments, edge assignments, and the smoothness of edge assignments, at all scales. The central conceptual motor in our constructions and estimates is a split up of the field at each length scale, locally, into a pure gauge field, and a deviation field. An example is presented establishing the general inevitability of gauge singularities, as a consequence of fall off requirements on the continuum field of an excitation.

1. Introduction

This is the fifth in a series of papers developing a treatment of the finite volume Euclidean Yang–Mills quantum field theory in four dimensions. To read this paper familiarity with the Papers I and IV in this series is assumed. (We refer to papers in this series with Roman numerals.) Not all of the material in these two papers will be of present interest to us, in particular the interesting mechanism of "gauge invariant coupling" introduced in IV will not be herein used. Of most interest will be Sects. 8–11 and the appendices of IV; we will detail references in the course of the paper. The abstract and introduction of IV serve also as a partial introduction to this paper (and the overall program) and should now be read; we in this introduction complement the introduction of IV. We begin by addressing two questions we have been often asked.

Does the procedure use a lattice formalism, or work with continuum fields? In the old days one asked, "is the electron a particle or a wave?" One there dealt with a "particle-wave duality." We deal with a "*lattice-continuum duality*," as

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