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Small Divisors with Spatial Structure in Infinite Dimensional Hamiltonian Systems

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Abstract. A general perturbation theory of the Kolmogorov-Arnold-Moser type is described concerning the existence of infinite dimensional invariant tori in nearly integrable hamiltonian systems. The key idea is to consider hamiltonians with a *spatial structure* and to express all quantitative aspects of the theory in terms of rather general weight functions on such structures. This approach combines great flexibility with an effective control of the various interactions in infinite dimensional systems.

1. Basic Concepts and Notions

The purpose of this paper is to present a perturbation theory for integrable hamiltonian systems of the Kolmogorov-Arnold-Moser type that comprises the classical result for general perturbations in the finite dimensional case [Kol, Arn-1, Mos-1, SM, Arn-2, Mos-3], finite chains of weakly coupled oscillators [VB, Way-3], finite dimensional systems with short range interactions [Way-1], systems of infinitely many oscillators with finite range couplings [FSW, VB], and a few more infinite dimensional systems with varying kinds of couplings and localizations. Indeed, our work was initiated and inspired by the progress in this area due to Bellissard, Fröhlich, Spencer, Vittot, and Wayne (in alphabetical order) and grew out of an attempt to obtain a unified approach to their results.

The key idea is to consider perturbations not as a single chunk but rather as composites of smaller pieces reflecting an underlying *spatial structure*. The allowable size of these pieces is determined by *weights* associated with their supports. These weights also determine all other quantitative aspects of the theory such as the shape of domains and the small divisor conditions. The validity of those nonresonance conditions is tied to some distribution property of the spatial structure with respect to the weight and cardinality of its components.

Spatial structures are characterized by a single structure property, and weight functions by the properties of monotonicity and subadditivity – see (1) and (2)

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