Commun. Math. Phys. 126, 575–595 (1990)

Communications in Mathematical Physics

Quantization on the Virasoro Group

Victor Aldaya[1,*] and José Navarro-Salas[2]

[1] The Blackett Laboratory, Imperial College, London SW7 2BZ, UK
[2] Departamento de Fisica Teórica, Facultad de Fisicas, Universidad de Valencia, Burjasot 46100-Valencia, Spain, and IFIC (Centro Mixto Universidad de Valencia-C.S.I.C.), Spain

Abstract. The quantization of the Virasoro group is carried out by means of a previously established group approach to quantization. We explicitly work out the two-cocycles on the Virasoro group as a preliminary step. In our scheme the carrier space for all the Virasoro representations is made out of polarized functions on the group manifold. It is proved that this space does not contain null vector states, even for $c \leqq 1$, although it is not irreducible. The full reduction is achieved in a straightforward way by just taking a well defined invariant subspace $\mathscr{H}_{(c,h)}$, the orbit of the enveloping algebra through the vacuum, which is irreducible for any value of c and h. $\mathscr{H}_{(c,h)}$ is a proper subspace of the space of polarized functions for those values of c and h for which the Kac determinant is zero. We give the local version of these group representations as well as the associated classical phase space structures, i.e., symplectic form and Noether invariants.

I. Introduction

The Geometric Quantization of Co-adjoint Orbits of a Lie group G [1,2] is intended to be the quantum mechanical description of a dynamical system defined by the group G itself. In GQCO the phase space M of the dynamical system is constituted by the symplectic manifold structure with which the coadjoint orbits are naturally endowed. The Hilbert space of wave functions is a space of sections of a line bundle on the symplectic manifold M once the so-called polarization (or Planck) conditions are imposed [1]. This scheme aims to establish in this way a correspondence between the set of irreducible unitary representations (quantizations) of G and that of its co-adjoint orbits. Nevertheless this association is in general not quite well achieved, as has been made evident when applied to the

Research partially supported by the Conselleria de Cultura de la Generalitat Valenciana, the Plan de formación del Personal investigador, the Comision Interministerial de Ciencia y Tecnologia (CICYT) and the British Council

* On leave of absence from 2