

The BRS Method and Geometric Quantization: Some Examples

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Abstract. We recall the method of quantization of constraints as outlined by Kostant and Sternberg. We discuss various examples to test compatibility between geometric quantization and this procedure. In almost all cases reduction and quantization commute.

0. Introduction

Kostant and Sternberg have proposed in their paper [KS] a very general scheme to quantize a reduced system by using BRS symmetry. However they make no choice of a specific quantization method. In this paper we test in some examples their procedure using geometric quantization.

Although the procedure itself can be stated in quite simple terms, we nevertheless give a lengthy theoretical survey because it contains a non trivial-modification of the Dirac quantization of constraints [Dir]. Moreover in the first part of our paper we point out a series of technical problems which can be related to the KS method.

Our motivation in dealing with this procedure is to see whether some problems and difficulties encountered in geometric quantization can be solved in this way—notably the problems found in the quantization of spin-spin interaction Hamiltonians and those related to the quantization of the geodesic flow. We should mention that our examples (and the “theory”) treat only the special case in which the reduction from the original (finite dimensional) phase space to the reduced one is given by a free action of a Lie group and in which the constrained manifold is coisotropic (only first class constraints). Our examples do not comply with the conditions found in Gotay’s paper [Got] where sufficient conditions are given for geometric quantization and reduction to commute. In particular we do not require

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