Commun. Math. Phys. 126, 469–482 (1990)

Communications in Mathematical Physics
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Entropy and Global Markov Properties

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Abstract. We extend, refine and give simple proofs of some recent results on the validity of global Markov properties for classical spin systems. One of the new results is that there is a global Markov property that is satisfied by equilibrium states in general. The proof of this establishes formulas for the entropy and free energy that show that these quantities are, for d-dimensional systems, given in terms of $(d-1)$-dimensional systems. Furthermore, we show that global Markov properties imply the absence of some types of symmetry breaking.

1. Introduction

Consider a classical spin system on $\mathbb{Z}^d$ with a translation invariant interaction-round-faces potential $\Phi = \{\Phi_X\}_{\substack{X \subset \mathbb{Z}^d \\ X \subset \text{a unit } d\text{-cube}}}$ on the configuration space $\Omega \equiv \Omega_0^{\mathbb{Z}^d}$, where Ω_0 is a finite set and $\Phi_X : \Omega_X \equiv \Omega_0^X \to \mathbb{R}$ are real functions. A state μ is a Gibbs state for the potential Φ if it satisfies the DLR equations,

$$\mu_\Lambda(\sigma_\Lambda | \sigma_{\Lambda^c}) = \frac{1}{Z_{\sigma_{\Lambda^c}}} \exp\left[-H_\Lambda(\sigma_\Lambda) - W_\Lambda(\sigma_\Lambda, \sigma_{\Lambda^c}) \right]$$

for each finite $\Lambda \subset \mathbb{Z}^d$, where Λ^c is the complement of Λ, W_Λ is the function on $\Omega_\Lambda \times \Omega_{\Lambda^c}$ defined by

$$W_\Lambda(\sigma_\Lambda, \sigma_{\Lambda^c}) = \sum_{\substack{X : X \cap \Lambda \neq \phi \\ X \cap \Lambda^c \neq \phi}} \Phi_X(\sigma_X),$$

and where $Z_{\sigma_{\Lambda^c}}$ is determined by the normalisation

$$\sum_{\sigma_\Lambda \in \Omega_\Lambda} \mu(\sigma_\Lambda | \sigma_{\Lambda^c}) = 1.$$