Commun. Math. Phys. 126, 133–156 (1989)

Communications in Mathematical Physics

Spectral Stability Under Tunneling

Ph. Briet**, J. M. Combes**, and P. Duclos**

Centre de Physique Théorique*, CNRS-Luminy, Case 907, F-13288, Marseille Cedex 09, France

Abstract. We study the spectral properties of multiple well Schrödinger operators on $\mathbb{R}^n$. We give in particular upper bounds on energy shifts due to tunnel effect and localization properties of wave packets. Our methods are based on Agmon type estimates for resolvents in classically forbidden regions and geometric perturbation theory. Our results are valid also for an infinite number of wells, arbitrary spectral type and in non-semi-classical regimes.

I. Introduction

The analysis of multiple well Schrödinger operators $H := -g^2\Delta + V$ on $\mathcal{H} := L^2(\mathbb{R}^n)$ is a new challenging aspect of the spectral theory for elliptic operators, linked to many problems of current interest like quasi-periodic potentials, composite media or disordered systems. It has attracted considerable attention in the last few years; for one dimensional problems let us mention in particular the work of Harrell [Ha1] using ODE methods, Jona-Lasinio, Martinelli and Scoppola [JMSc1] using stochastic mechanics and Combes, Duclos and Seiler [CDS1], where the use of perturbation methods were initiated. In dimensions larger than one there is in particular a remarkable amount of results from Helffer and Sjöstrand [HSj1–3] and Simon [Sim1–3]. These works concern the semi-classical regime for regular positive potentials with finitely many compact wells, having non-degenerate minima (see however [MaR] and references therein for results in the degenerate case). In such situations the leading asymptotics of energy shifts for the lowest eigenvalues under tunneling can be derived rigorously. One natural question is whether one can also obtain rigorous non-asymptotic estimates for arbitrary potentials; in other words is it possible to obtain general stability conditions both for the spectrum (upper bounds on energy shifts) and the localisation of wave functions. This is the problem which is investigated in this

* Laboratoire Propre, Centre National de la Recherche Scientifique
** Phymat, Université de Toulon et du Var