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Bound on the Mass Gap for Finite Volume Stochastic Ising Models at Low Temperature

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Abstract. We consider a sequence of finite volume $\Lambda \subset \mathbf{Z}^d$, $d \geqq 2$, reversible stochastic Ising models in the low temperature regime and having invariant measures satisfying free boundary conditions. We show that associated with the models are random hitting times whose expectations, regarded as a function of Λ, grow exponentially in $|\Lambda|^{(d-1)/d}$; moreover, the mass gaps for the models shrink exponentially fast in $|\Lambda|^{(d-1)/d}$. A geometrical lemma is employed in the analysis which states that if a Peierls' contour is sufficiently small relative to the faces of Λ, then the fraction of the contour tangent to the faces is less than a constant smaller than one.

1. Introduction

Let Ω_Λ be the semigroup generator for a finite volume $\Lambda \subset \mathbf{Z}^d$ ($\mathbf{Z}^d$ is the d-dimensional integer lattice, $d \geqq 2$) stochastic Ising model defined by [cf. 1, 2]

$$\Omega_\Lambda f(\sigma) = \sum_{i \in \Lambda} c_\Lambda(i, \sigma) \partial_i f(\sigma). \tag{1.1}$$

Here, σ is an Ising configuration with $\sigma(i) = \pm 1$, f is an arbitrary real-valued function of the configuration, and $\partial_i f(\sigma) = f(\sigma_i) - f(\sigma)$ with σ_i the new spin configuration obtained from σ by flipping σ at site i, i.e., $\sigma_i(j) = \sigma(j)$ $j \neq i$, $\sigma_i(i) = -\sigma(i)$. For the sake of definiteness, we will assume throughout that the speed functions are given by

$$c_\Lambda(i, \sigma) = \exp\left(-\frac{\beta}{2} \partial_i H_\Lambda(\sigma)\right) \tag{1.2}$$

with H_Λ the usual nearest neighbor Ising Hamiltonian,

$$H_\Lambda(\sigma) = -\sum_{\substack{i, j \in \Lambda \\ \langle i, j \rangle = 1}} \sigma(i)\sigma(j), \tag{1.3}$$

although certainly other choices of Hamiltonian and speed functions are possible. We will moreover assume that Λ is a (hyper)-cube, although here again, the