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Conformal Scalar Fields and Chiral Splitting on Super Riemann Surfaces *

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Abstract. We provide a complete description of correlation functions of scalar superfields on a super Riemann surface, taking into account zero modes and non-trivial topology. They are built out of chirally split correlation functions, or conformal blocks at fixed internal momenta. We formulate effective rules which determine these completely in terms of geometric invariants of the super Riemann surface. The chirally split correlation functions have non-trivial monodromy and produce single-valued amplitudes only upon integration over loop momenta. Our discussion covers the even spin structure as well as the odd spin structure case which had been the source of many difficulties in the past. Super analogues of Green's functions, holomorphic spinors, and prime forms emerge which should pave the way to function theory on super Riemann surfaces. In superstring theories, chirally split amplitudes for scalar superfields are crucial in enforcing the GSO projection required for consistency. However one really knew how to carry this out only in the operator formalism to one-loop order. Our results provide a way of enforcing the GSO projection to any loop.

1. Introduction

Closed fermionic strings are built out of independent left and right movers on the world-sheet. In the Ramond-Neveu-Schwarz formulation, the string coordinates are the world-sheet scalars x^μ and Majorana spinors ψ^μ_-, ψ^μ_+, and although both are space-time vectors, the theory actually carries space-time fermions [1]. A supersymmetric spectrum is obtained after performing the projection of Gliozzi-Scherk-Olive (GSO), which retains only the even G-parity states in the Neveu-Schwarz sector, and the positive (space-time) chirality states in the Ramond sector [2]. On world-sheets of non-trivial topology, spinors such as ψ^μ_-, ψ^μ_+ require a spin structure – which specifies the sign ambiguity as the spinor is transported around a

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