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Hydrodynamics of Stochastic Cellular Automata*

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Dedicated to Roland Dobrushin

Abstract. We investigate a stochastic version of cellular automata used for simulating hydrodynamical flows, e.g. the HPP and FHP models. The extra stochasticity consists of "random exchanges" between neighboring cells which conserve momentum. We prove that, in suitable limits, these models satisfy the appropriate continuous Boltzmann and hydrodynamic equations, the same as those conjectured for the original models (except that there is no negative viscosity contribution). The results are obtained by proving a very strong form of propagation of chaos and by using Hilbert-Chapman-Enskog type expansions. Explicit proofs are presented for the stochastic HPP model.

1. Introduction and Results

Computer simulations on hydrodynamic cellular automata reproduce patterns observed in real physical fluids [1–4]. This is, at first sight, remarkable, even astonishing, since at a microscopic-particle level the dynamics seems at best a caricature of the interactions between real molecules. The explanation lies in the observation, basic to the development of these automata, that the macroscopic behavior of a fluid does not depend on the detailed features of the particle interactions: systems which microscopically look very different may give rise to the same type of macroscopic behavior.

Scale separation is responsible for this behavior and the purpose of the present paper is to rigorously prove its occurrence in a stochastic variant of these models. As

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