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# Anderson Localization for the 1-*D* Discrete Schrödinger Operator with Two-Frequency Potential

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*Dedicated to Roland Dobrushin*

**Abstract.** We prove the complete exponential localization of eigenfunctions for the 1-*D* discrete Schrödinger operators with quasi-periodic potentials having two basic frequencies. It is shown also that for such operators there is no forbidden zones in the spectrum, unlike the operators with one basic frequency.

## 1. Introduction

The phenomenon of the Anderson localization, or exponential decay of eigenfunctions of random self-adjoint operators, has been studied very intensively during the last several years. It is fairly clear now that basic mechanisms of the Anderson localization are essentially the same for differential operators like the Schrödinger operator

$$H_\varepsilon(\alpha) = -\varepsilon \frac{d^2}{dx^2} + V(x,\alpha)\,, \tag{1}$$

$\alpha$ being a random parameter, and for their discrete analogues like

$$(H_\varepsilon(\alpha)\psi)(n) = \varepsilon(\psi(n-1)+\psi(n+1)) + V(n,\alpha)\psi(n)\;. \tag{2}$$

Furthermore, the localization is shown both for "true random" and for almost periodic potentials $V(\cdot,\alpha)$. From the formal point of view, both classes can be considered in a more general context of random potentials having the form

$$V(x,\alpha) = F(T^{-x}\alpha)\;, \qquad \alpha\in\Omega\;, \qquad T^x:\Omega\to\Omega\;,$$

where $\{T^x, x\in\mathbb{Z}\}$ or $\{T^x, x\in\mathbb{R}\}$ is a group of automorphisms of a probability space $(\Omega,\mu)$ and $F:\Omega\to\mathbb{R}$ is a measurable function. But it is worth emphasizing that such a general approach is not just a formality. On the contrary, it turns out that the language of operator ensembles is adequate for the investigation of a very important phenomenon like tunneling in disordered media due to long-range resonances.

Historically, the Anderson localization was initially proven for random operators whose coefficients at different points were independent or very weakly