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The Large-Scale Limit of Dyson's Hierarchical Vector-Valued Model at Low Temperatures. The marginal case $c=\sqrt{2}$

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Dedicated to Roland Dobrushin

Abstract. In this paper we construct the equilibrium states of Dyson's vector-valued hierarchical model with parameter $c=\sqrt{2}$ at low temperatures and describe their large-scale limit. The analogous problems for $\sqrt{2}<c<2$ and $1<c<\sqrt{2}$ were solved in our papers [1] and [2]. In the present case the large-scale limit is similar to the case $\sqrt{2}<c<2$, i.e. it is a Gaussian self-similar field with long-range dependence in the direction orthogonal to and a field consisting of independent Gaussian random variables in the direction parallel with the magnetization. The main difference between the two cases is that now the normalizing factor in the direction of the magnetization contains, beside the square-root of the volume, a logarithmic term too.

1. Introduction

First we briefly describe the model we are investigating. Dyson's hierarchical model is a one-dimensional classical spin model on the lattice $\mathbf{Z}=\{1,2,\ldots\}$. Its Hamiltonian function depends on a parameter a, $1<a<2$, and is defined as

$$\mathscr{H}(\sigma)=-\sum_{i\in\mathbf{Z}}\sum_{\substack{j\in\mathbf{Z}\\ j>i}} d(i,j)^{-a}\sigma(i)\sigma(j), \tag{1.1}$$

where $d(i,j)=2^{n(i,j)-1}$, and

$$n(i,j)=\min\{n, \text{ there exists some } k \text{ such that } (k-1)2^n<i,\, j\leqq k2^n\}.$$

We are dealing with vector-valued models, where $\sigma(j)\in R^p$ with some $p\geqq 2$. If $x\in R^p$ and $y\in R^p$ then xy denotes scalar product. We consider models with the free measure ν,

$$\frac{d\nu}{dx}(x)=p_0(x)=p_0(x,t)=C(t)\exp\left\{-\frac{x^2}{2}-\frac{t}{4}|x|^4\right\}, \qquad x\in R^p, \tag{1.2}$$