Commun. Math. Phys. 124, 543–552 (1989)

Communications in Mathematical Physics

Singularity of the Density of States for One-Dimensional Chains with Random Couplings

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Abstract. We prove that the density of states for the tight-binding model with off-diagonal disorder under general conditions diverges for $R \to 0$ at least as $\sim \frac{1}{|E|(\ln|E|)^4}$. This result is established through the study of the recurrence properties of an associated Markov chain.

1. Introduction

In the field of Schrödinger equations with random potential (see [S] and references therein) a great deal of attention has been given to the study of the properties of the density of states as a function of the energy E. For lattice systems with diagonal disorder Wegner ([W]) proved that if the distribution of the potential is absolutely continuous with bounded density, then the integrated density of states is absolutely continuous and its derivative $\rho(E)$ is bounded. With more detailed assumptions and/or in one dimension one can prove smoothness properties of the density of states a) in any dimension at high disorder ([CFS, BCKP]); b) in one dimension at any disorder by using transfer matrix methods ([ST, CK]). In [CK] it is proven that for a wide class of distributions for the potential including non-absolutely continuous ones the density of states is smooth in one-dimension.

The case with only off-diagonal disorder, i.e. when only the couplings between different sites are random, presents some different features. Here Wegner's result does not apply. Indeed Dyson ([Dy]) found a one-dimensional model whose density of states can be exactly computed and displays a singularity of the type $\frac{1}{|E|(\ln|E|)^3}$ as $E \to 0$. It has been argued that this singularity should not be particular to this model. In [ER] and [TC] numerical computations and heuristic arguments are presented to support this claim.

* Partial financial support by GNAFA (CNR)

** Partial financial support by CNPq, grant n.303795-77FA