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p-Adic Quantum Mechanics

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Abstract. An extension of the formalism of quantum mechanics to the case where the canonical variables are valued in a field of *p*-adic numbers is considered. In particular the free particle and the harmonic oscillator are considered. In classical *p*-adic mechanics we consider time as a *p*-adic variable and coordinates and momentum or *p*-adic or real. For the case of *p*-adic coordinates and momentum quantum mechanics with complex amplitudes is constructed. It is shown that the Weyl representation is an adequate formulation in this case. For harmonic oscillator the evolution operator is constructed in an explicit form. For primes p of the form $4l+1$ generalized vacuum states are constructed. The spectra of the evolution operator have been investigated. The *p*-adic quantum mechanics is also formulated by means of probability measures over the space of generalized functions. This theory obeys an unusual property: the propagator of a massive particle has power decay at infinity, but no exponential one.

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