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A Unified Approach to Phase Diagrams in Field Theory and Statistical Mechanics

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Abstract. We construct the phase diagram of any system which admits a low-temperature polymer or cluster expansion. Such an expansion turns the system into a hard-core interacting contour model with small, but not necessarily positive, activities. The method uses some of Zahradnik's ideas [Z1], but applies equally well to systems with complex interactions. We give two applications. First, to low-temperature $P(\phi)_2$ models with complex couplings; and second, to a computation of asymptotics of partition functions in periodic volumes. If the index of a supersymmetric field theory is known, the second application would help determine the number of phases in infinite volume.

1. Introduction

In many systems in statistical mechanics and quantum field theory, the problem of competing ground states arises, even when the parameters of the model permit weak- or strong-coupling expansions. The standard example of such a system is the N-state Ising model at low temperatures (at or near first-order phase transitions.) In the absence of a symmetry between the states, one is faced with the problem of determining which states are thermodynamically stable. The successful theory of Pirogov and Sinai [PS] was developed to determine the stable phases of systems such as the N-state Ising model.

Subsequent authors developed the method for applications in more complicated systems. Quantum field theory models involving continuous space-time and continuous spins were handled by [I]. Statistical mechanics models with nontrivial structure within each phase were treated by [BKL, DZ], and others. In these studies, the need to use certain probability arguments from [PS] was a heavy burden. Especially in field theory, the combined requirements of decoupling and positive probability measures necessitated a very complicated procedure. Complex interactions were not accessible at all.

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