Commun. Math. Phys. 122, 527–529 (1989)

Communications in Mathematical Physics

Erratum

The Quantum Theory of Second Class Constraints: Kinematics

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Commun. Math. Phys. **119**, 75 (1988)

In the proof of Lemma 3.2, [1], we proved that if for all $U \in \langle \mathscr{U} \rangle$ of the form $U = \sum_{\ell} \lambda_\ell U_\ell$, $U_\ell \in \langle \mathscr{U} \rangle$ we have $\sum \lambda_\ell = 1$, then $\mathbb{1} \notin C^*(\mathscr{U} - \mathbb{1})$. This proof is wrong because not all positive elements $A \in C^*(\mathscr{U})_+$ can be written as $A = B^*B$ with $B = \sum \lambda_\ell U_\ell$, though the set of such elements is of course dense in $C^*(\mathscr{U})_+$. Hence the proven inequality $\omega(B^*B) \geqq 0$ is not sufficient to ensure that ω is positive, so it does not follow that ω extends from the $*$-algebra generated by $\mathscr{U}$ to $C^*(\mathscr{U})$. In fact we know that in general this part of Lemma 3.2 is wrong:

Assertion. *There is no general condition on the $*$-algebra generated by a set of constraints $\mathscr{U}$ which is equivalent to $\mathbb{1} \in C^*(\mathscr{U} - \mathbb{1})$.*

Proof. We exhibit a $*$-algebra $\mathscr{K}$ containing a group of unitaries $\mathscr{U}$, and complete it in two different C^*-norms. In one of the resulting C^*-algebras we will have $\mathbb{1} \in C^*(\mathscr{U} - \mathbb{1})$, and in the other $\mathbb{1} \notin C^*(\mathscr{U} - \mathbb{1})$.

Let G be a discrete group acting on a unital C^*-algebra $\mathscr{F}$ with the action $\alpha : G \mapsto \mathrm{Aut}\,\mathscr{F}$. Construct $\tilde{\tilde{\mathscr{F}}} := M(G_\alpha \times \mathscr{F})$ which contains $\mathscr{F}$ and a faithful unitary representation $U : G \mapsto \tilde{\tilde{\mathscr{F}}}_u$ of G which implements α, i.e. $\alpha_g = \mathrm{Ad}\, U_g$. So U_G is a group.

Lemma 1. *$U_G \subset \tilde{\tilde{\mathscr{F}}}$ is a linearly independent set.*

Proof. $(U_g f)(r) := \alpha_g(f(g^{-1}r))\ \forall g, r \in G$, $\forall f \in \ell^1(G, \mathscr{F})$, where $\ell^1(G, \mathscr{F}) := \left\{ f : G \mapsto \mathscr{F} \,\middle|\, \sum_{g \in G} \|f(g)\| < \infty \right\}$. Assume U_G is linearly dependent, i.e. $\exists \beta_k \in \mathbb{C} \backslash 0$, $g_k \in G$ all different and $N < \infty$ such that $\sum_{k=1}^{N} \beta_k U_{g_k} = 0$. Hence $\forall f \in \ell^1(G, \mathscr{F})$ we have $\sum_{k=1}^{N} \beta_k \alpha_{g_k}(f(g_k^{-1}r)) = 0$. Choose $f(r) := \mathbb{1}\delta(r, e)$. Then $\sum_{k=1}^{N} \beta_k \mathbb{1} \delta(g_k^{-1}r, e) = 0\ \forall r \in G$, so for $r = g_k$ this implies $\beta_k = 0$, which contradicts our assumption. $\square$

Take $\mathscr{U} = U_G$ for the chosen constraint set, let the $*$-algebra $\mathscr{K}$ be generated by U_G, hence it is the linear space generated by U_G. Let the C^*-algebra $\mathscr{A}$ be the C^*-