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Deformations of Super-KMS Functionals*

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Abstract. We investigate the stability of the super-KMS property under deformations. We show that a family of continuous deformations of the super-derivation in the quantum algebra yields a continuous family of deformed super-KMS functionals. These functionals define a family of cohomologous, entire cocycles.

I. Introduction

In this paper we investigate the super-KMS (sKMS) property of functionals ω on a quantum algebra. Our interest in sKMS functionals was inspired by work of Kastler and by conversations with Alain Connes [C1, K, JLO2]. The sKMS construction relies on the existence of a super-derivation d acting on a dense subalgebra of a C^*-algebra $\mathscr{A}$. The square of d is the infinitesimal generator of a continuous, one-parameter automorphism group α_t of the quantum algebra. The usual KMS property relates the cyclicity of a state ω to the analytic continuation of a group α_t of automorphisms. The sKMS property also involves invariance under the super-derivation d whose square generates the automorphism group α_t.

It is known that an sKMS functional on a quantum algebra defines an entire cyclic cocycle τ. This is just the Chern character which Jaffe, Lesniewski, and Osterwalder defined in the context of supertrace functionals on a quantum algebra [JLO1]. The sKMS property ensures that the functional ω – and the cocycle τ which is derived from it – are invariant under this group action.

In this paper we study the stability of this structure under perturbations of d. We study only bounded perturbations which arise from the graded (super) commutator with an odd element q of the algebra $\mathscr{A}$. We show that such perturbations d_q of d can be used to define a deformation ω^q of ω which satisfies the sKMS property. Furthermore, the corresponding family of cocycles τ^q are cohomologous. Of course, more singular (unbounded) perturbations can lead to

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