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Embeddings of $U(1)$-Current Algebras in Non-Commutative Algebras of Classical Statistical Mechanics*

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Abstract. Motivated by multiplicative K-homology, and understanding critical phenomena in some classical statistical mechanical models, we construct actions of $GL(\infty)$ on the operator algebras of V. Jones and Ocneanu, and analyse these in terms of embeddings of $U(1)$-current algebras.

1. Introduction

In this paper we shall describe a construction of actions of the group $GL(\infty)$ on the V. Jones and Ocneanu algebras, to which both authors of this paper arrived at independently and for different reasons.

The first author was motivated by the search for a multiplicative analogue of the known additive K-homology theory. In this the Jones' index of subfactors would be the multiplicative analogue of the codimension of a subspace of a Hilbert space. This analogy would be in the same way as the classification of outer conjugacy classes of automorphisms of the hyperfinite factor of type II_1 is the multiplicative analogue of the classification of unitaries modulo the compact operators. The basic notion of a Fredholm module $(\mathfrak{h}, D)$ on an algebra $\mathfrak{A}$ gets replaced by a pair (A, σ_t) where A is a C^*-algebra, σ_t a one parameter group of automorphisms, while the unitary group $\mathfrak{U}(\mathfrak{A})$ acts on A by automorphisms. It is easy to get such a multiplicative module out of a Fredholm module $(\mathfrak{h}, D)$ by the CAR construction, and the possibility of replacing the CAR algebra by the algebras of V. Jones and Ocneanu served as one motivation for the search of an action of $GL(\infty)$.

The second author was motivated by statistical mechanics, in particular by the transfer matrix method, and the problem of understanding critical phenomena and the continuum limit in classical models such as those of Potts and of Andrews et al.

The transfer matrix method allows us to reduce a two-dimensional classical or commutative statistical mechanical model to a one-dimensional quantum or non-

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