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Convergent Renormalization Expansions for Lattice Gauge Theories

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Abstract. In this paper we introduce an inductive description of the complete effective densities including large field domains, and we show that the renormalization transformations preserve the form of the densities. This completes the renormalization group analysis for superrenormalizable models and yields convergent expansions in this case.

0. Introduction

In this paper we continue our study of the renormalization group analysis of the non-Abelian lattice gauge field theories. In the previous papers, referred to as [I, II], we have analyzed the renormalization transformations in the small field regions. The basic goal here is to do such an analysis for the complete model, without any restrictions on field variables, i.e. including all large field regions. Thus we study the complete renormalization transformations T, without any restrictions on integration variables. Let us recall that they are of the form

$$(T\varrho)(V)=\int dU\delta(\bar{U}V^{-1})\varrho(U) \ , \tag{0.1}$$

where $\varrho(U)$ is a function of the gauge field variables U on a lattice T, $\bar{U}$ is the averaged field on the lattice $T^{(1)}$, and V is a new gauge field on the lattice $T^{(1)}$. The averaging operation is defined by the formula (0.4), or (0.12) in [I], but we may take any averaging operation satisfying several general properties, analogous to the properties (0.5)–(0.9) [I]. We have to apply an additional operation after each renormalization transformation T. This operation is denoted by $\mathbf{R}$, and it changes effective densities on large field regions. Thus, we construct a sequence of effective densities $\{\varrho_k\}$ by applying successively the operations $\mathbf{R}T$ to the initial density $\varrho_0=\exp[-(1/g_0^2)A-E]$, where A is the Wilson action, and E is a normalization

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