Commun. Math. Phys. 117, 431–439 (1988)

Communications in Mathematical Physics

Predecessor States for Certain Cellular Automata Evolutions

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Abstract. This paper reports the solution of the problem of finding which inputs for a function of discrete variables will result in any specified output for the set of binomially determined nearest neighbor additive cellular automata defined on finite or half-infinite sequences. In computing the solution to this problem a process which is the discrete analog to backward integration is defined. This process is determined in terms of an operator which exhibits an interesting period multiplying property.

I. Introduction

A cellular automaton consists of a finite or infinite lattice with values at each lattice site drawn from a discrete set, together with a deterministic local rule which specifies the site values at time $t+1$ in terms of site values at time t. They were initially introduced by Von Neumann [1] in an attempt to find simple mathematical systems which exhibited some features of life. Applications of cellular automata have been made in modeling heart fibrillation [2]; as parallel processors [3,4]; as prime number sieves [5]; in image processing and pattern recognition [6,7]. More recently, work of Wolfram [8,9,10] has stimulated a renewal of interest (see, e.g., papers in Physica 10D) and it has been suggested [11] that cellular automata may have a major role to play "complexity engineering;" i.e., in the design of complex systems which exhibit specific properties.

Before such applications can be realized, however, there are several problems which must be solved. One of these is the predecessor problem [11]: is it possible to determine the predecessor states for any given state of an arbitrary cellular automata? As Wolfram points out, there may be no general solution to this problem as it is, in the general case, NP-complete.

This paper presents the solution to the predecessor problem for the special case of binomially determined nearest neighbor additive cellular automata defined over Z_p, where p is prime. Although this is a limited subset of the set of all cellular automata, the solution presented is significant for several reasons: it is a first step in attack on the general case; it is a direct analog of backwards integration; an interesting period multiplying property emerges; and, most of the current results appearing in the literature are based on analysis of additive cellular automata—