Commun. Math. Phys. 117, 349–351 (1988)

Communications in
Mathematical
Physics

Comment

Global Anomalies on Orbifolds

Daniel S. Freed[1] and Cumrun Vafa[2]

[1] Department of Mathematics, University of Chicago, Chicago, IL 60637, USA
[2] Department of Physics, Harvard University, Cambridge, MA 02138, USA

Commun. Math. Phys. **110**, 349–389 (1987)

In this brief addendum we correct an error in our paper and report a solution to some open questions.

Unfortunately, Proposition 7.6 is incorrect. The mistake comes in our definition of the map $\phi_{\mathbb{R}}$. Although it is well-defined, it is not a homomorphism, hence not a representation. Thus we cannot see the characteristic classes of representations of the space group G as preimages of characteristic classes of representations of the point group P.

A simple example illustrates the problem. Let G be the infinite dihedral group. It is an extension of $\mathbb{Z}/2\mathbb{Z}$ by the integers, with the nontrivial element in $\mathbb{Z}/2\mathbb{Z}$ acting by reflection $x \mapsto -x$. Now construct a 2 dimensional complex representation by letting the generator of the integers act by the matrix $\begin{pmatrix} \exp(it) & 0 \\ 0 & \exp(-it) \end{pmatrix}$ and the reflection act by $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Then if t is irrational this representation does not factor through any finite group.

On the other hand, in any specific example it is usually easy to reduce the calculation of w_2 and λ to a representation of a finite group, in view of Proposition 7.8.

The representation "ϕ_P" in Corollary 7.7 should be changed to "ϕ".

We also warn the reader that although Proposition 7.3 is correct, it is possible for $v(\varrho_P)$ to be nonzero and $v(\varrho)=j^*v(\varrho_P)$ to vanish.

Similarly, the argument in Sect. 6, which also purports to reduce space group anomalies to point group anomalies, is correct only when the Wilson lines can be continuously turned off. This does not happen in general, as the previous example illustrates. However, if we can continuously change the gauge group imbedding of the space group to an imbedding in a finite group, then again the vanishing of the first Pontrjagin class in the group cohomology of the finite group will be sufficient to guarantee the absence of anomalies to all loops. In all the examples we know this can be done, and is equivalent to level matching condition for each element of the space group in the case of an abelian point group. Examples of this type can be